

Scaling of cosmic string loops

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(Received 6 December 2005; revised manuscript received 20 June 2006; published 28 September 2006)

We study the spectrum of loops as a part of a complete network of cosmic strings in flat spacetime. After a long transient regime, characterized by production of small loops at the scale of the initial conditions, it appears that a true scaling regime takes over. In this final regime the characteristic length of loops scales as $0.1t$, in contrast to earlier simulations which found tiny loops. We expect the expanding-universe behavior to be qualitatively similar. If this expectation is correct, then the large loop sizes have important cosmological implications. In particular, the nucleosynthesis bound then becomes $G\mu \lesssim 10^{-7}$, much tighter than that obtained from earlier analyses.

DOI: [10.1103/PhysRevD.74.063527](https://doi.org/10.1103/PhysRevD.74.063527)

PACS numbers: 98.80.Cq, 11.27.+d

Cosmic strings could be formed as linear defects at symmetry breaking phase transitions in the early universe [1]. Alternatively, they could arise as fundamental or D -strings at the end of brane inflation [2–4]. Strings can produce a variety of observational effects, and searches are now underway for their signatures in gravitational lensing, CMB anisotropies, and gravitational wave background. A good theoretical understanding of string networks is crucial for interpreting the results of these searches. However, despite much effort, the evolution of cosmic strings is not yet fully understood.

An evolving string network consists of two components: long strings and subhorizon closed loops. It is fairly well established that long strings exhibit scaling behavior: both the average distance between the strings $d(t)$ and the coherence (or persistence) length $\xi(t)$ scale with the cosmic horizon,

$$d(t) \sim \xi(t) \sim t. \quad (1)$$

In the early work on cosmic strings, it was expected [5,6] that the typical length of closed loops scales in a similar manner, $l(t) \sim t$. The first numerical simulations of string evolution seemed to support this scenario [7]. However, later, high-resolution simulations [8,9] showed that the loop sizes were actually much smaller than the horizon and gave no evidence for scaling. On the contrary, the typical loop size remained nearly constant and close to the resolution limit of the simulations. The simulations also revealed that long strings had a significant substructure, with short-wavelength wiggles all the way down to the resolution limit.

The standard scenario of string evolution that has emerged from these findings assumes that the typical size of loops $l(t)$ is set by the scale of the smallest wiggles, which is in turn determined by damping due to gravitational radiation [10]. The typical loop size is then given by

$$l(t) \sim \alpha t \quad (2)$$

with [11] $\alpha \sim (G\mu)^\gamma$, where G is Newton's constant and μ is the mass per unit length of string. With plausible assumptions about the spectrum of wiggles, the power index γ is in the range $2 \leq \gamma \leq 3$. The observational bound on the string mass parameter $G\mu$ is $G\mu < 10^{-6}$, and the corresponding bound on α is $\alpha < 10^{-12}$, indicating that the loops are extremely small.

A more radical string evolution scenario, proposed in [12], suggests that the loops are even smaller. It claims that strings lose most of their energy by direct emission of microscopic loops of size not much greater than the string thickness. This idea was not confirmed in other simulations [13,14]. It is hard to tell which, if any, of these scenarios is correct, since the loop sizes they suggest are well beyond the resolution limits of current simulations.

To address this issue, we developed in [15] a flat-space string simulation which does not suffer from the problem of smallest resolution scale. This simulation uses functional forms for the string positions and is exact to the limits of computer arithmetics. In Ref. [15] we used our simulation to show that the spectrum of wiggles on long strings scales with time, even in the absence of gravitational damping. The spectrum has a universal form; it is peaked at $l \sim 0.3t$ and declines slowly towards smaller scales.

In the present paper we report on the study of the closed loop component of the simulation. We find that loops are produced in a wide range of sizes, with most of the string length going into relatively large loops of size $l \sim 0.1t$, comparable to the interstring distance. This scaling behavior is established only after a long transient regime, characterized by copious production of tiny loops whose size is set by the scale of the initial string network. We believe that it was this transient regime that was observed in earlier simulations.

The numerical results presented here are based on the computer simulation described in [15]. In order to be able to study the evolution at late times we periodically increase the simulation volume as described in [15]. Specifically,

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we create 8 identical replicas of the simulation box and glue them together to form a box twice the size of the original one. The resulting unphysical correlations on superhorizon scales are removed by allowing the string reconnection probability to be $p < 1$. We used the value of $p = 0.5$ which maximizes the rate of decay of the correlations.¹ We have verified that simulations with $p = 0.2$ and 0.8 give similar results.

Until recently, there was a widespread opinion that all values of p other than $p = 1$ are unphysical and therefore of little interest. However, it was pointed out in [3,16,17] that for fundamental and D -strings p generally differs from 1, and may even be $\ll 1$. The dependence of network evolution on the value of p is an important problem, but we will not try to address it here. It will require more extensive simulations, since the evolution is very slow for small p .

To start our simulation we generate four different Vachaspati-Vilenkin initial conditions [18] and overlay them in the same simulation box. We displace the four realizations relative to each other so that equivalent lattice points lie on the corners of a tetrahedron of height 0.5. In previous simulation work we found that the correlation length $\xi(t)$ is larger than the interstring distance $d(t)$ by a factor of about 2.6 in the scaling regime, while in the initial conditions this factor is only about 1.3. By overlaying four initial conditions, we decrease the initial interstring distance by a factor 2, while not changing the correlation length, so the initial conditions have properties more similar to a scaling regime.

The maximum initial box size with four interlaced initial conditions that we can simulate on a single processor with 3GB of memory is 50, which becomes 800 by time 1000, after 4 doublings of the box size. The results presented here are the averages of 25 simulations. The expansion of the simulation volumes takes place at typical times 33, 93, 259, and 924.

A snapshot of a cubic section of the network at time 800 can be seen in Fig. 1. There are some long strings crossing the cube, many small loops, and a few loops of sizes up to 60, comparable to the average interstring separation at that time.

The spectrum of wiggles, as defined in [15], is shown in Fig. 2 as a function of kt . There are two peaks present on the plot: one scaling and one nonscaling. The nonscaling peak slowly decays and moves to large values of kt , but stays roughly at the same value of k that corresponds to the initial correlation length. In contrast, the scaling peak remains at almost the same value of kt . Note that in [15] the nonscaling peak was completely eliminated by smooth-

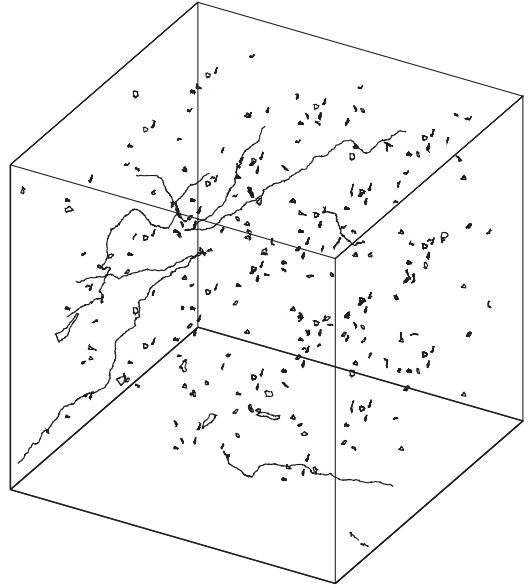


FIG. 1. A 150^3 section of the network at time 800. Loops of sizes less than 10 are not shown.

ing. We have not used smoothing here, because it potentially distorts the spectrum of small loops and because the jumps it introduces make it difficult to see trends in the loop production spectrum.

We characterize the rate of loop production by the function $n(l, t)$ —the number of loops produced per unit loop length per unit volume of the network per unit time. In a scaling network, the number of loops with sizes between l and $l + dl$ produced in a volume L^3 evolving from time t to $t + dt$ is the same as the number with sizes between $2l$ and $2l + 2dl$ produced in a volume $8L^3$ evolving from time $2t$ to $2t + 2dt$. Thus, for scaling,

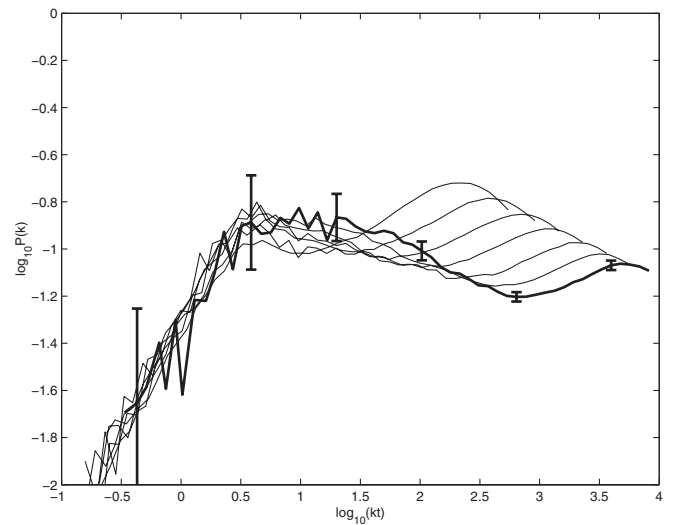


FIG. 2. Evolution of the power spectrum $P(kt)$ vs kt at times: 60, 100, 160, 250, 400, 600, 900 (bold). Error bars are the one sigma run-to-run variation.

¹To minimize disturbance to the string network we do not use the smoothing procedure of [15], and we keep the reconnection probability always 0.5, rather than switching between 0.5 and 1. Apart from making the time dependence smoother, this has no significant effect on the results.

$$n(l, t) = t^{-5}f(x), \quad (3)$$

where f can be any function of $x = l/t$.

In a cosmological string network, infinite strings self-intersect and produce loops. These loops can either reconnect with other strings, fragment further by self-intersections, or oscillate without self-intersections. But in a simulation, all strings are closed, so we need some definition of the point at which a loop is considered to have been produced. We proceed as follows.

First, we say that a loop is a survivor if neither it, nor any fragment produced from it, rejoins any other string. To conserve computer memory, we do not allow strings shorter than a minimum length κt to rejoin the network, so any string smaller than κt is automatically a survivor. We have verified in [15] that the network evolution is rather insensitive to the choice of κ , and is not significantly modified even if one sets $\kappa = 0$. In the present simulation we used $\kappa = 0.25$.

A loop is primary if it is a survivor but none of its ancestors are survivors. In Fig. 3 we plot the primary loop production function $x^2 f_p(x)$, where $f_p(x) = t^5 n_p(l, t)$ and n_p is the production function of primary loops with the same conventions as for n . The graph thus shows the fraction of total length produced in primary loops for each logarithmic interval in x .

For a scaling spectrum, we expect $f_p(x)$ to be independent of time. Instead, we see two peaks closely related to the two peaks in the power spectrum. The scaling peak (on the right) does not move in x but remains at

$$l_p \sim 0.3t \quad (4)$$

and increases in amplitude. On the other hand, the non-scaling peak (on the left) always remains at the scale of the

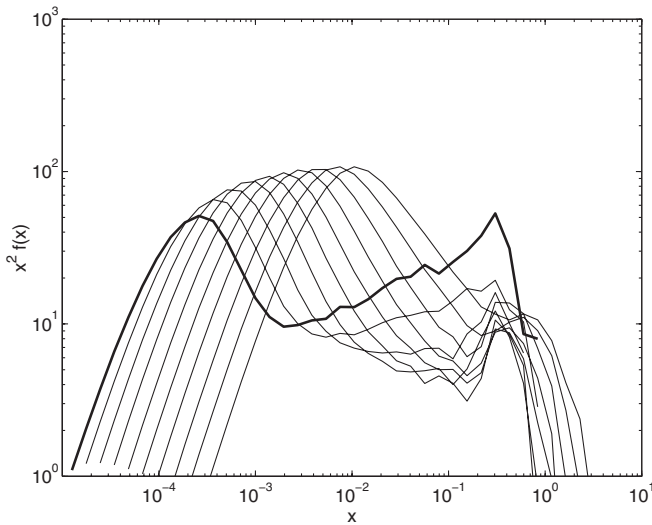


FIG. 3. The primary loops production function $x^2 f_p(x)$ in equal logarithmic bins of time centered at times 64, 89, 125, 175, 245, 342, 479, 671, and 939 (bold).

initial correlation length, so it moves to smaller values of x , and decreases in amplitude. At early times the production of loops is dominated by short lengths, but as long strings become smoother, the loop production at small scales decreases. In contrast, the scaling peak is subdominant in the beginning, but as the time advances it steadily grows. At even later times, we expect the peak related to initial conditions to vanish, leaving a scaling spectrum.

A primary loop of size l is likely to fragment in time $t \sim l/4$. The fragmentation process continues until all loops find themselves in non-self-intersecting trajectories. In Fig. 4 we plot the final production function of loops $x^2 f(x)$. As for primary loops, there are two peaks in the final loop production function: one scaling and one non-scaling. The nonscaling peak is going down and we expect it eventually to vanish, while the scaling peak slowly becomes the dominant one and remains at constant x ,

$$l_f \sim 0.1t. \quad (5)$$

The nonscaling peak in Fig. 4 is present only because of the production of small primary loops shown in Fig. 3. If one considers only final loops produced from primary loops whose length is greater than 8.4, the small-scale peak is absent, as shown in Fig. 5. ($l = 8.4$ is about twice the size of the smallest loops in the initial string network. This particular choice is due to the logarithmic binning we used for the loop statistics.)

The scaling part of the final loop production function can be fit by a power law

$$f(x) \approx Ax^{-\beta}, \quad (6)$$

with

$$\beta = 1.63 \pm 0.03. \quad (7)$$

We found $A = 82 \pm 2$ in our simulations, but since a large

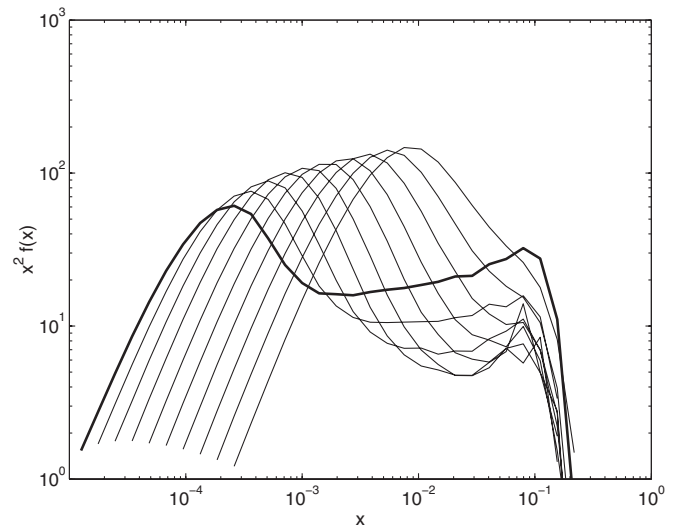


FIG. 4. The final production function of loops $x^2 f(x)$ for the same time intervals as Fig. 3.

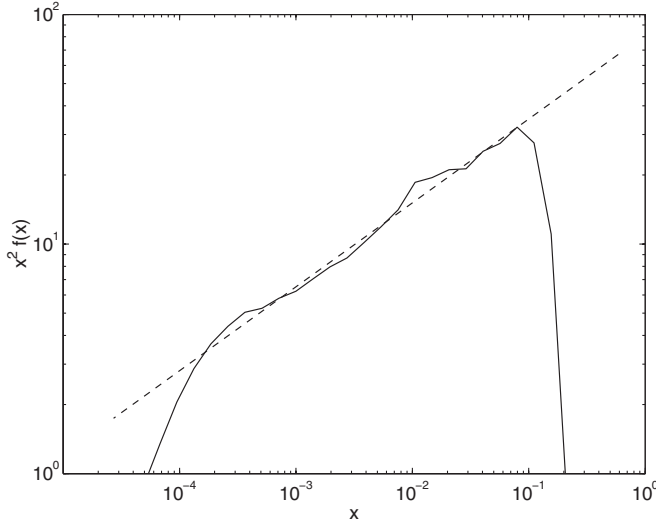


FIG. 5. The final production function of loops $x^2 f(x)$ from the last time interval in Fig. 4, considering only primary loops longer than 8.4.

part of the loop production is still at small sizes even at the last times simulated, we expect the final scaling value of A to be somewhat larger. (The error bars above indicate only statistical variation between different runs of the simulation.)

The corresponding loop production rate is:

$$n(l, t) = t^{-5} f(x) \approx A t^{\beta-5} l^{-\beta}, \quad (8)$$

with a sharp cut-off for large loops at $l \sim 0.1t$. From Eq. (7), the total number of loops produced is divergent, while their total length is finite.

Our simulations are done only in flat space, but one can make a reasonable conjecture about the expanding universe. The expansion of the universe stretches the excitations on the strings, so small-scale structure tends to be reduced relative to the flat-space case. Thus we expect the nonscaling peak at the initial condition size to be eliminated more quickly, and the loop production spectrum to fall more rapidly toward small sizes in the expanding universe than in flat space. If as a result the power index β is decreased by at least 0.63, Eq. (8) will give a convergent total number of small loops.

In any case, we expect, on the basis of the simulations described here, that most of the energy produced in loops from a cosmic string network appears at scales comparable to the interstring distance. This agrees with early expectations [5,6], but not with later simulations [8,9].

If there is a scaling process of loop production in an expanding universe, then loops produced over time at some fixed fraction α of the horizon size give rise to a spectrum of presently existing loops [19]

$$N(l, t) \propto l^{-\beta_c} \quad (9)$$

where $\beta_c = 5/2$ for a radiation-dominated universe and $\beta_c = 2$ for matter. This spectrum rises more steeply toward smaller l than the loop production function, Eq. (6), indicating that late loop production does not significantly affect the form of the spectrum, Eq. (9). Thus as long as we know that $\beta < \beta_c$ in the expanding universe, we do not need to know the precise value of β . The form of the loop distribution is given by Eq. (9). This distribution is cut off at small scales by gravitational back reaction.

The large value of α suggested by our simulations implies a higher energy density of the string-generated gravitational wave background and tighter observational constraints on the string parameter $G\mu$. The requirement that gravitational waves from strings do not affect the predictions of big bang nucleosynthesis can be expressed as [19] (for $\alpha \gtrsim 10^2 G\mu$)

$$G\mu \lesssim 10^{-8} \alpha^{-1}. \quad (10)$$

It follows from the analysis in [20] that for $\alpha \gtrsim 10^2 G\mu$ the millisecond pulsar observations yield the bound $G\mu \lesssim 10^{-7}$, which is similar to (10) for $\alpha \sim 0.1$. (Both of these bounds assume that the reconnection probability is $p \sim 1$. We expect p to produce a denser string network and so make this bound more stringent. Unfortunately simulation of small p is difficult because the network evolves much more slowly.)

Recent work [21] by Ringeval, Sakellariadou and Bouchet (RSB) discussed loop production in expanding-universe simulations. They found that the distribution of loops grows steeply toward small scales, with the power index $\beta \approx 3.0$ in the radiation era and $\beta \approx 2.5$ in the matter era. Since these indices are larger than β_c , the index β in their loop production function must also have these values, in sharp contrast with our result, Eq. (7). The most likely explanation of this discrepancy is that the RSB simulation is still in the transient regime dominated by small loop production. Indeed, the shape of the loop production function obtained by RSB is similar to that in our Fig. 4 at times $t \lesssim 100$, when the scaling peak is not yet pronounced.

This work was supported in part by the National Science Foundation under grants 0353314 and 0457456.

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