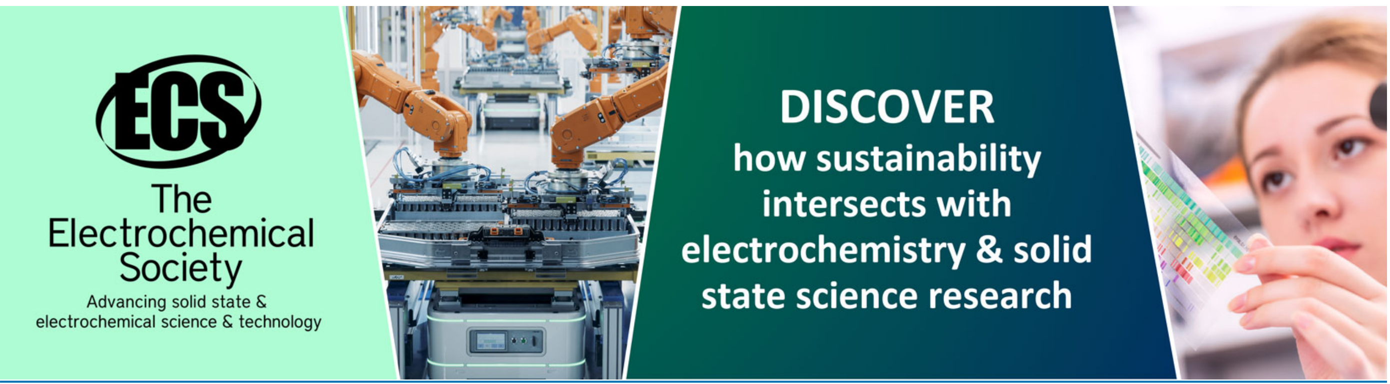
Journal of Physics: Conference Series

Fermion Zero Modes and Topological-charge on a Domain Wall of the D-brane-like Dot

To cite this article: Ikuzo Kanazawa 2015 *J. Phys.: Conf. Ser.* **592** 012135



Journal of Physics: Conference Series **592** (2015) 012135 doi:10.1088/1742-6596/592/1/012135

Fermion Zero Modes and Topological-charge on a Domain Wall of the D-brane-like Dot

Ikuzo KANAZAWA[1]

[1]Department of Physics, Tokyo Gakugei University, Tokyo、Japan

E-mail: `kanazawa@u-gakugei.ac.jp`

Abstract. Anomalous excitations in the electron state of self-assembled InAs quantum dots have been analyzed from viewpoint of field-theoretical formula. It is suggested strongly that exotic excitations with fractional charges might exist in the semiconductor-dot with the domain wall shell. We have discussed the toy-model of the quantum-dot, which is composed of three degenarated D4-brane-like solitons.

1. Introduction

Self-assembled quantum dots(SAQD) have been intensively investigated because of their potential applications in many optoelectronic devices as well as from a fundamental physics point of view [1,2]. Due to relatively strong confinement in lateral direction, SAQD can be regarded as artificial atoms. In most experiments reported so far, SAQD have been investigated by optical spectroscopy, in particular photoluminescence(PL). PL experiments on single dots are well established, which overcome the inhomogeneously broadened line widths in typical ensemble measurements [3]. With resonant inelastic scattering, one has direct access to the electronic elementary excitations of low-dimansional electron system, the so-called spin-density and charge-density excitations. Brocke et al.[4] have very interesting results in InAs SAQD by resonant inelastic light scattering. By applying a gate voltage between a metallic front gate and a back electrode, they can charge the quantum dots with single electrons. With resonant inelastic light scattering, they have observed directly the elementary electronic excitations of the few-electron quantum-dot atoms, which are formed by the SAQD. They have observed excitations, which might be identified as transition of electrons from the s-shell to the p-one (s-p transition) and from the p-shell to the d-one (p-d transition) of the quasi-atoms. Taking into account the effect of Coulomb interaction, they explain the shift and broadeing of the s-p transition of collective excitations as additional excitations at low energies. But the origin of additional excitations is not confirmed so far. Recently the present author has reported the importantce of the hole-induced domain-wall in magnetoresistance in diluted magnetic semiconductors [5,6] and doped manganites [7,8]. In this study, we discuss an amormalous excitations in the quantum dot, which might correspond to exotic excitations with fractional charges, extending the previous formula [9-11] and Callen-Harvey theory [12]. In addition, we discuss the toy-model of the quantum-dot, which is composed of three degenerated D4-brane-like solitons.

2. A model system

Many polymers such as polyacetylene exhibit degenarate ground states with the ensuing domain wall(soliton) confugurations in quasi-(1+1) dimensional system. Su, Schrieffer, Heeger [20] have shown that domain wall solitons in polyacetylene display anomalous quantum numbers, which are a signature of fermion fractionization. Generally, the appearance of a fractional fermion nuber is understood as an effect of vacuum polarization,i.e. a modification of the fermionic Dirac sea by its interaction with solitons. The magnitude of the ground state fermion number depends in a nontrivial way on the fermion content of the theory and details of interactions between fermions and topological background fields. The fractional part of the fermion number associated with a soliton configuration is invariant under local deformation of the bosonic background fields. Brocke et al.[4] observed the shift and broadening of the s-p transition of collective excitations in InAs SAQD by resonant inelastic light scattering. An interesting point is that the shift and broadening effects are enhanced when the Fermi energy level is located near the p-shell energy level. The p-shell energy level will be distorded strongly near the domain wall shell. It is likely that the crossing between the p-shell energy level and the Fermi energy level may occur near the domain wall shell. In this case, we assume that the low-energy excitations are described approximately in terms of massive relativistic fermions (with a mass, m, of the order of the energy gap). Thus we will introduce the effective Lagrangian L_{eff} as follows,

$$L_{eff} = \overline{\Psi}(i\gamma_i\partial_i - \phi_1(r) - i\gamma^5\phi_2(r))\Psi$$

Ψ is the Fermi wavefunction. $\phi_1(r)$ and $\phi_2(r)$ are the bosonic background fields, and γ^5 is chirality matrix. The Dirac Hamiltonian is as follows,

$$H = \begin{pmatrix} \phi_1 & -\partial_1 + \phi_2 \\ \partial_1 + \phi_2 & -\phi_1 \end{pmatrix}$$

$= a_\pm = m$, The background fields $\phi_1(r)$ and $\phi_2(r)$ are the mass and the background field with domain wall shell, respectively. $\phi_2(r) \longrightarrow +v$ when $r \gg r_0$. r_0 is the radius of the dot. $\phi_2(r) \longrightarrow -v$ when $r_0 \gg r \sim 0$. Using the theoretical formula [12], the zero mode is given by

$$\Psi_0 = \chi_0(r)exp[-\int_0^r \phi_2(z)\,dz], \tag{1}$$

with $i\gamma_i\partial_i\chi_0 = 0$ so that the zero modes are solutions of the massless Dirac equation on the domain wall shell. The background fields $\phi_1(r)$ and $\phi_2(r)$ are the constant function,m , and the arbitrary functions of r with the limits,

$$\phi_1 \begin{pmatrix} r \gg r_0 \\ r_0 \gg r \sim 0 \end{pmatrix}$$

$= a_\pm = m,$

$$\phi_2 \begin{pmatrix} r \gg r_0 \\ r_0 \gg r \sim 0 \end{pmatrix}$$

$= b_\pm = \pm v$. We introduce the parameter τ and extend $\phi_1(r)$ and $\phi_2(r)$ to functions of (r, τ). We assume that both $\phi_1(r \gg r_0, \tau = -\infty) = a_+ = m$, $\phi_2(r_0 \gg r, \tau = -\infty) = b_- = -v$. So that the comparison Hamiltonian H_0 is

$$H_0 = \begin{pmatrix} a_+ & -\partial_1 + b_- \\ \partial_1 + b_- & -a_+ \end{pmatrix}$$

Using the Mellin transformation of the odd spectral density [10], the Fermi number N corresponding to the Hamiltonian H is given by

$$\begin{aligned} N &= -\frac{1}{2}\eta_H = T_{3+1} + Index(D) \\ &+ \frac{1}{2\pi}[arctan[\upsilon/m] - arctan[-\upsilon/m]] \\ &= T_{3+1} + Index(D) + \frac{1}{\pi}\theta. \end{aligned} \tag{2}$$

$$\theta = arctan[\frac{\upsilon}{m}]. \tag{3}$$

η_H is the η - invariant term of the Atiya-Patodi-Singer index theorem [13]. Index (D) is the index of the Dirac operator D. T_{3+1} is the Pontryagin index of the background fields with the domain wall shell. The fractional part of the fermion number in eq.(2) is $\frac{1}{\pi}\theta$. This suggests strongly that excitations with the fractional charges exist in this dot-system. It is likely that these exotic excitations with the fractional charges might create new energy-levels in the dot. In addition, it is expected that the present model will be applicable to the dot of a narrow-gap semiconductor such as PdTe [11]. Now, we shall consider the quantum-dot from the viewpoint of the soliton such as $D4$-branes [14,15]. The $D4$-brane(a soliton), given by the one-pole 't Hooft ansatz [16], $\rho(r) = 1 + \lambda^2/|x-a|^2$ with width λ, position a=0, and $x = (x_1, x_2, x_3)$ generates a Skyrme field [17] of the hedgehog form $U(x) = exp[if(r)x \cdot \tau]$ with a profile function given by

$$f(r) = \pi[1 - (1 + \lambda^2/r^2)^{-1/2}]. \tag{4}$$

Here we shall consider the toy-model of a quantum-dot, which is composed of three degenerated $D^{i=1,2,3}4$-branes with the profile f(r). It is assumed that a width of the quantum-dot is $\sim \lambda$. From three degenerated $D^{i=1,2,3}4$-branes [18], it is known that the SU(3) gauge fields are created among different D^i4-branes. As a consequence of ground-state degenerary of three degenereted D^i4-branes, stable elementary excitations of fractional charges $\pm(1/3)$lel and $\pm(2/3)$lel can be created [9,19-21] in the toy-model, which is composed of three degenerated D^i4-branes. Because relativistic field-theory of meson and hadrons is very complex, one cannot help but notice the analog with the quark model. If it is assumed that "i" of D^i4-branes correspond to the colours, the SU(3) gauge fields are analogous to the gluon fields. In the trapping of electron-particle as lepton in the quantum-dot, the excitations with +(2/3)lel and -(1/3)lel in three degenerated D^i4-branes are analogous to the up-quark and down-quark, respectively. In the trapping of μ-particle as lepton in the quantum-dot, the excitations with +(2/3)lel and -(1/3)lel in three degenerated D^i4-branes are analogous to the charm-quark and strange-quark, respectively. In the trapping of τ-particle as lepton in the quantum-dot, the excitations with +(2/3)lel and -(1/3)lel in three degenerated D^i4-branes are analogous to the top-quark and botom-quark, respectively. Because the present model of a quantum-dot is too simple and the toy-model, the present model might not correspond to the real states in mesons and hadrons. However, thses analogies are very fascinating.

3. Conclusion

We have analyzed anomalous excitations in the electron state of self-assembled quantum semiconductor-dot from viewpoint of field-theoretical formula. It is reported that exotic excitations with fractional charges might exist in the semiconductor-dot with the domain wall shell. We have discussed the toy-model of the quantum-dot, which is composed of three degenerated $D^{i=1,2,3}$4-branes-like solitons.

References

[1] Marzin J A et al. 1994 *Phy. Rev. Lett* **73** 716
[2] Drexler H et al. 1994 *Phy. Rev. Lett* **73** 2257
[3] Gammon D and Steel D G 2002 *Phys. Today* **55** 36
[4] Broche T et al. 2004 *Physica* **E 22** 478
[5] Kanazawa I 2006 *Phys.Lett* **A 355** 460
[6] Kanazawa I 2007 *Physica* **E 40** 277
[7] Kanazawa I 2010 *Phys. Status Solids* **B 247** 644
[8] Kanazawa I 2013 *J. Low Temp. Phys.* **172** 177
[9] Goldstone J and Wilczek F 1981 *Phys. Rev. Lett.* **47** 986
[10] Niemi A 1985 *Nucl. Phys.* **B 253** 14
[11] Fradkin E et al. 1986 *Phys. Rev. Lett.* **57** 2967
[12] Callen Jr C G and Harvey J A 1985 *Nucl. Phys.* **B 250** 427
[13] Atiyah M, Patodi V, and Singer I 1973 *Bull. London. Math. Soc.* **5** 229
[14] Polchinski J 1995 *Phys. Rev. Lett.* **75** 4724
[15] Sakai T and Sugimoto S 2005 *Prog. Theor. Phys.* **113** 847
[16] 't Hooft G 1976 *Phys. Rev.* **D 14** 3432
[17] Atiyah M F and Manton N S 1986 *Phys. Lett.* **B 222** 438
[18] Polchinski J 1998 String Theory I, II, Cambridge Univ. Press.
[19] Jackiw R and Rebbi C 1976 *Phys. Rev.* **D 13** 3398
[20] Su W P, Schriefer J R, and Heeger A J 1979 *Phys. Rev. Lett.* **42** 1698
[21] Su W P and Schriefer J R 1981 *Phys. Rev. Lett.* **46** 738