A Discussion on Deformation Parameter Features

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The Uncertainty Principle (HUP), originating from Heisenberg's seminal paper in 1927, delves into the intricacies of measurement processes within Quantum Theory. One well-known illustration of this is the Heisenberg microscope argument, which attempts to determine the position of an electron using photons. Initially, gravitational interactions between particles were disregarded due to the apparent weakness of gravity compared to other fundamental interactions. However, over time, HUP evolved into a fundamental theorem within the framework of Quantum Mechanics. Despite its foundational status, discussions about fundamental aspects of nature necessitate considering gravity. This inclusion has unfolded over decades, from early attempts to generalize HUP to recent propositions such as those from string theory, deformed special relativity, and black hole physics. Several revisions of the classical Heisenberg argument have been proposed. For instance, one version suggests that a beam of photons with energy E can detect an object of size δx, roughly given by $\delta x \approx \frac{\hbar c}{2E}$. This implies that higher energies allow for the exploration of smaller details. If one accounts for the formation of micro black holes in high-energy scatterings, with a gravitational radius roughly proportional to the scattering energy, the uncertainty relation needs modification. The modified relation becomes

$$\delta x \approx \frac{\hbar c}{2E} + \beta {l_p}^2 \frac{2E}{\pi \hbar c} \tag{1}$$

where β is a dimensionless parameter. Although the precise value of β is not determined by theory, it is generally assumed to be around unity, as suggested by some models of string theory. An analytic calculation of β has supported this assumption. However, efforts have been made to experimentally constrain β. This modification leads to a Generalized Uncertainty Principle (GUP), expressed as

$$\Delta x \Delta p \geq \frac{\hbar}{2} \left[1 + \beta \left(\frac{\Delta p}{m_p c} \right)^2 \right] \tag{2}$$

in which Δx and Δp are uncertainties in position and momentum respectively, and m_p is the Planck mass. For mirror-symmetric states, the GUP is equivalent to the commutator

$$[\hat{x}, \hat{p}] = i\hbar\left[1 + \beta\left(\frac{\hat{p}}{m_p c}\right)^2\right] \tag{3}$$

The GUP finds application in various domains such as quantum mechanics, quantum field theory, thermal effects in QFT, and modifications of quantization rules.

A Quantum Mechanics view about Deformations

Over the past decade, there has been a vigorous discourse surrounding the quantifiable attributes posited by different types of Generalized Uncertainty Principles (GUPs). This debate has primarily revolved around the anticipated modifications and their experimental implications. Notable among the proposed experiments are those devised by the Brukner and Marin groups. The exploration of the dimensionless deforming parameter β of GUP can be roughly categorized into two groups. In the first group, researchers such as Kempf, Mann, Brau, Vagenas, and Nozari, have translated GUP into a deformed commutator and subsequently developed a deformed quantum mechanics framework. This deformed commutator, generally expressed as

$$[\hat{X}, \hat{P}] = i\hbar\left(1 + \beta\frac{\hat{P}^2}{m^2 c^2}\right) \tag{4}$$

where the fundamental variables $\hat{X}$ and $\hat{P}$ are envisioned as high-energy operators applicable at or near the Planck scale. These operators exhibit nonlinear representations, denoted as $\hat{X} = X(\hat{x})$ and $\hat{P} = P(\hat{p})$, in terms of the usual position and momentum operators $\hat{x}$ and $\hat{p}$ at low energies, obeying the standard Heisenberg commutator

$$[\hat{x}, \hat{p}] = i\hbar \tag{5}$$

This methodology is typically employed to deduce constraints on β originating from non-gravitational factors by analyzing well-established physical phenomena using the new variables $\hat{X}$ and $\hat{P}$, and comparing the results with experimental data. The explicit calculations hinge upon the specific transformation

$\hat{X} = X(\hat{x})$ and $\hat{P} = P(\hat{p})$, which generally involves nonlinear and non-canonical characteristics, as the commutator $[\hat{x}, \hat{p}]$ does not equal $[\hat{X}, \hat{P}]$, indicating that the corresponding Poisson brackets are not preserved.Researchers compute corrections to quantum mechanical forests, such as energy shifts in the hydrogen atom spectrum, the Lamb shift, Landau levels, Scanning Tunneling Microscope observations, charmonium levels, etc. While the bounds on β derived from this approach are notably stringent, a contentious issue arises regarding the potential dependence of the anticipated shifts on the specific representation of variables X and P in the fundamental commutator.

Gravitational bounds on β by the Classical Mechanics

Some texts approach that modify classical mechanics by introducing deformations to standard formulations, often resembling quantum mechanical features. One such approach involves deforming Newtonian mechanics through adjustments to the Poisson brackets, resembling quantum commutators. This modification is represented by the relation

$$[\hat{x}, \hat{p}] = i\hbar(\ +\beta_0 \hat{p}^2) \tag{6}$$

which transforms to

$$\{X, P\} = (1 + \beta_0 P^2) \tag{7}$$

The parameter β_0 is defined as

$$\beta/(m_p^2 c^2) \tag{8}$$

Researcher utilize this deformed mechanics to compute the precession of Mercury's perihelion, interpreting it as an additional effect to the precession predicted by General Relativity (GR). Comparing this result with observations yields a very precise agreement, leading to an extremely tight constraint of $\beta < 10^{-66}$. However, a critique of this approach arises from its linear superposition with GR, neglecting potential modifications to GR at order β. This omission raises questions about the coexistence of GR and GUP-modified Newtonian mechanics and how their respective precession errors combine. Furthermore, as β approaches 0, the model only recovers Newtonian mechanics, necessitating the addition of GR corrections as an independent structure. Consequently, the physical validity of this approach and the resulting bound on β remain dubious.

Another approach introduces a covariant formalism to deform classical Poisson brackets, leading to a β-deformed geodesic equation and violation of the Equivalence Principle. Interestingly, this violation solely stems from the deformed Poisson brackets and not from general covariance or modifications to GR equations or solutions. For instance, deforming Poisson brackets akin to quantum commutators results in modified equations of motion for a particle in a Newtonian potential. This modification leads to violations of the Equivalence Principle, showcasing that even within simple Newtonian mechanics, deviations arise from the altered brackets. Despite this, the Ghosh formalism retains covariance as β approaches 0, restoring standard GR predictions in this limit. Thus, it was predicted a bound on β of $\beta < 10^{21}$ from the violation of the universality of free fall, indicative of a (weak) equivalence principle violation. Tkachuk explores the equivalence principle within deformed classical mechanics by considering composite bodies. It has been proposed that kinetic energy remains independent of the body's composition but relies solely on its total mass, introducing deformation parameters, β_{0_i} for each constituent particle. This construction allows for the recovery of the equivalence principle within deformed classical mechanics, albeit with each particle possessing its unique minimal length. This feature conflicts with the universality of gravitation, as it implies different minimal lengths for different particles, contrary to the universally constant Planck length.

Calculation of Deformation Parameter

One of the aims is computing an exact value of β by employing methods. It starts by introducing a generic deformation of the spherically symmetric metric, expressed as

$$F(r) = 1 - \frac{2GM}{rc^2} + \varepsilon\phi(r) \tag{9}$$

$$r_H = a - \frac{\varepsilon a\phi(a)}{1 + \varepsilon[\phi(a) + a\phi'(a)]} \tag{10}$$

with a denoting $2GM/c^2$. Subsequently, the deformed standard Hawking temperature, denoted as T, is calculated, incorporating the deformed metric function $F(r)$, yielding a comprehensive expression accounting for quantum corrections. From the respective first-order terms of these expansions, the exact value of β is extracted, expressed as

$$\beta = \frac{4\pi}{2m_p^2}[2\epsilon(a) + a\epsilon'(a)] \tag{11}$$

This extraction method is demonstrated through the consideration of leading quantum corrections to the Newtonian potential, investigated by Duff and Donoghue. Donoghue's approach reformulates General Relativity as an effective field theory, highlighting that at ordinary energies, gravity behaves akin to a well-behaved Quantum Field Theory (QFT). Quantum corrections at low energy, coupled with dominant effects at large distances, are attributed to the propagation of massless particles (gravitons). This leads to the derivation of an effective quantum-corrected gravitational potential, which encompasses both classical and quantum effects. Further, it can be discussed the definition of an effective Newtonian potential for a given metric, as well as the reverse process of mimicking a prescribed Newtonian potential with a corresponding metric. Utilizing these concepts, the metric imitating the quantum-corrected Newtonian potential is established. By identifying the correction term $\epsilon(r)$ within this context eventually leads to the determination of β. Notably, β is found to be approximately of order 1, corroborating similar findings from alternative methodologies.